BOSTON UNIVERSITY
GRADUATE SCHOOL

Thesis

MODERN SPECTROMETERS: THEIR THEORY AND FIELDS OF APPLICATION

by
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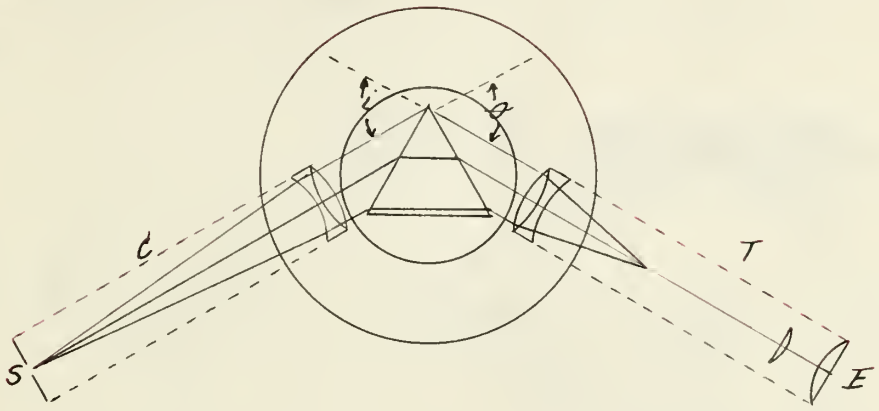
MODERN SPECTROMETERS: *******THEIR THEORY AND FIELDS OF APPLICATION

A. The Prism Spectroscope.

Diagram:



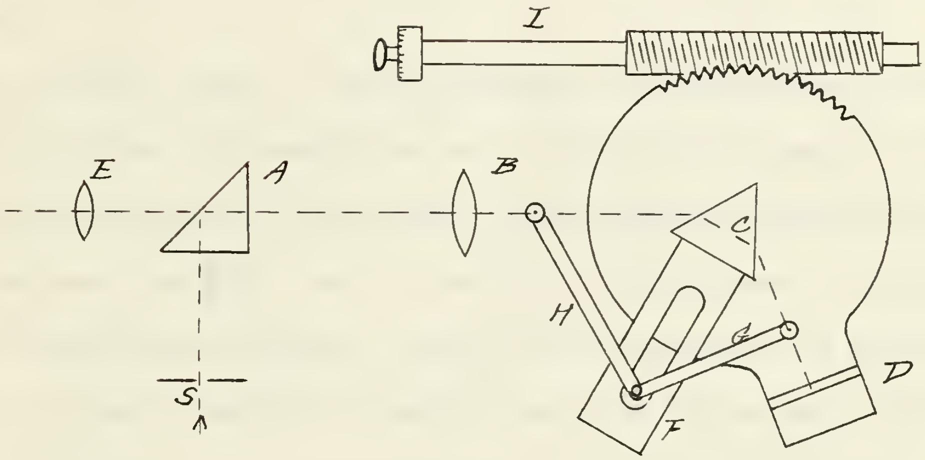
General description and theory:

A spectroscope is used for the purpose of facilitating a study of spectra. A narrow adjustable slit, which is strongly illuminated by the light to be examined, is used as a source. The slit is used as a source. The slit is used in order to produce a pure spectrum without overlapping. This slit is located at one end of a closed tube, called a collimator; at the other end is a telescope. A parallel beam of light is produced by the collimator. A prism is so placed, that these rays will fall on one of its faces. The rays after passing through the prism are received by the telescope, and are brought together to form a spectrum. This instrument is called a spectrometer when provided with a graduated circle for measuring the angular deviation of the light; a constant deviation prism spectrometer, when provided with a constant deviation prism, which is rotated by means of a drum graduated to read directly in wave

length. The Littrow type of spectrometer is a special type of spectroscope which because of its importance is described below.

The Littrow Type of Spectroscope

Diagram:



General description and theory:

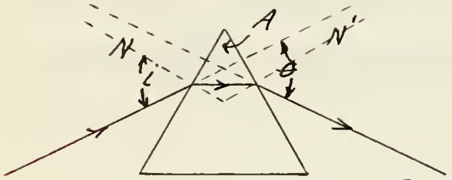
The light enters the slit S, and then is totally reflected by the prism A; the lens B directs it as a parallel beam upon the prism C, this transmits the rays at minimum deviation and they then fall normally upon the plane mirror D, which reflects them back through the prism to the lens B, where they pass to the eyepiece E. The instrument is so adjusted that the rays on the return journey pass above or below the right angle prism A. At F is shown an arrangement for automatically keeping the prism in the position of minimum deviation; the tie G is pivoted upon the arm carrying the mirror D, which is free to rotate around a vertical axis directly under the center of the prism C. The tie is pivoted upon the stand of the apparatus; both the ties G & H are pivoted together to a pin working in a slot in the arm F which carries the prism C. In order to cause

the spectrum to move across the field of view it is only necessary to rotate the arm carrying the mirror D, through a certain angle; then the prism C will move through half that angle, and therefore, will be automatically kept in the position of minimum deviation.

Dispersion

The refractive index of any transparent body depends upon the wave length of the light used. The way in which the refractive index μ varies with changing wave length $d\lambda$ is termed the dispersion and may be expressed as $\frac{d\theta}{d\lambda}$, that is, the ratio of the change in deviation to the change in wave length. The dispersion depends in a spectroscope upon the number and the refracting angles of the prisms employed, and also upon the nature of the medium out of which the prisms are made. The derivation of the value for the dispersion of a prism follows below:

With the refractive index given for a prism as



$$\mu = \frac{\sin \frac{A+\theta}{2}}{\sin \frac{A}{2}}$$

Differentiating $\frac{\partial \theta}{\partial \mu} = \frac{2 \sin \frac{A}{2}}{\cos \frac{A+\theta}{2}}$ but $\frac{A+\theta}{2} = \sin i$

Therefore $\frac{\partial \theta}{\partial \mu} = \frac{2 \sin \frac{A}{2}}{\cos i} = \frac{2 \sin \frac{A}{2}}{\sqrt{1 - \sin^2 i}}$

But $\sin i = \mu \sin \frac{A}{2}$

So that $\frac{\partial \theta}{\partial \mu} = \frac{2 \sin \frac{A}{2}}{\sqrt{1 - \mu^2 \sin^2 \frac{A}{2}}}$

which gives $\frac{\partial \theta}{\partial \mu}$ in terms of angle of the prism A, and the index of refraction.

Now from Hartman's formula

$$\mu = \mu_o + \frac{c}{(\lambda - \lambda_o)^a} \quad \text{and} \quad \lambda = \lambda_o + \frac{c}{(\mu - \mu_o)^{\frac{1}{a}}}$$

now giving a the value 1 (it is 1.2 for glass)

$$\mu = \mu_o + \frac{c}{\lambda - \lambda_o}$$

$$\lambda = \lambda_o + \frac{c}{\mu - \mu_o}$$

Differentiating $\frac{\partial \mu}{\partial \lambda} = -\frac{c}{(\lambda - \lambda_o)^2}$

And from above $\frac{\partial \theta}{\partial \mu} = \frac{2 \sin \frac{A}{2}}{\sqrt{1 - \mu^2 \sin^2 \frac{A}{2}}}$

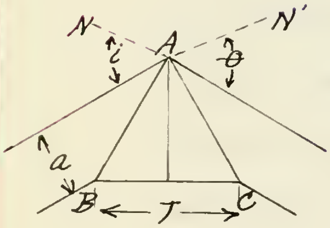
And $\frac{d\theta}{d\lambda} = \frac{\partial \theta}{\partial \mu} \times \frac{\partial \mu}{\partial \lambda} = \text{Dispersion}$

Resolving Power

It may be shown that with a prism the resolution depends entirely upon the size of the base, the angle of refraction having no influence. Therefore, all the possible prisms upon the same base give equal resolution. The value of the resolving power of a prism may be expressed as $\frac{\lambda}{d\lambda}$. The derivation for this expression is presented below:

$$a = AB \cos i = AB \sqrt{1 - \sin^2 i}$$

but $\sin i = \mu \sin \frac{A}{2}$



Therefore $a = AB\sqrt{1 - \mu^2 \sin^2 \frac{A}{2}}$ #1

Also $\frac{t}{2} = AB \sin \frac{A}{2}$

$t = 2AB \sin \frac{A}{2}$ #2

and dividing #2 by #1 $\frac{t}{a} = \frac{2 \sin \frac{A}{2}}{\sqrt{1 - \mu^2 \sin^2 \frac{A}{2}}}$

but from dispersion $\frac{\partial \theta}{\partial \mu} = \frac{2 \sin \frac{A}{2}}{\sqrt{1 - \mu^2 \sin^2 \frac{A}{2}}}$

So $\frac{\partial\theta}{\partial\mu} = \frac{t}{a}$ and $\partial\theta = \frac{t\,\partial\mu}{a}$ #3

Now Raleigh gives the expression $\frac{\mathcal{E}}{f} = \frac{\lambda}{a}$ and $\frac{\mathcal{E}}{f} = d\theta$

Therefore $d\theta = \frac{\lambda}{a}$ #4

So from #3 and #4 $d\theta = \frac{\lambda}{a} = \frac{t\,\partial\mu}{a}$

Multiplying by $\frac{a}{d\cancel{\lambda}}$; $\frac{a\,d\theta}{d\lambda} = \frac{t\,\partial u}{d\lambda} = \frac{\lambda}{d\lambda}$

Applications:

(a) The wave length of light may be calculated from the angle of deviation of the light. If possible, it is better always to work with the prism in the position of minimum deviation for every ray examined. In actual observations the fixed pointer in the eye piece is first focussed, and then, after focussing the image of the slit directly, without the intervention of the prism, the position of the telescope is read upon the divided circle; the telescope is then moved until the pointer is adjusted exactly on the spectrum line, when the position is again read upon the divided scale. In order to find the wave length of an unknown line it is possible to use an interpolation formula, such as Hartman's, from which the wave length of an unknown line can be obtained, if its deviation and the deviations and wave lengths of lines on each side of it be known. However, by far the best method is that of graphical interpolation; that is, by drawing the dispersion curve of the instrument; then it is only necessary to measure the deviation and the wave length may be read directly from the curve.

Applications continued:

(b) To measure (1) the angle of minimum deviation of light; (2) the refractive angle of the prism; and (3) the index of refraction of glass.

(1) To measure the angle of minimum deviation: using sodium light place the prism on the table end. Then rotate the table and find the position at which the image of the slit reverses its direction of rotation while the table is being rotated continuously in one direction. This is the position of minimum deviation. Having set the cross hairs accurately on the line in its position of minimum deviation, read the divided circle. Turn the prism on the table so that it produces deviation on the opposite side and when the prism is adjusted for minimum deviation, set the telescope on the same sodium line as before. The angle between these two positions is twice the angle of minimum deviation.

(2) To measure the refractive angle of the prism: Turn the prism with its defracting edge towards the collimator, dividing the beam of light, part falling on one face of the prism and part on the other. Set the telescope on one side of the image of the slit and read the divided circle; and then turn the telescope to the other side and repeat the above. The angle through which the telescope has turned is twice the refracted angle of the prism.

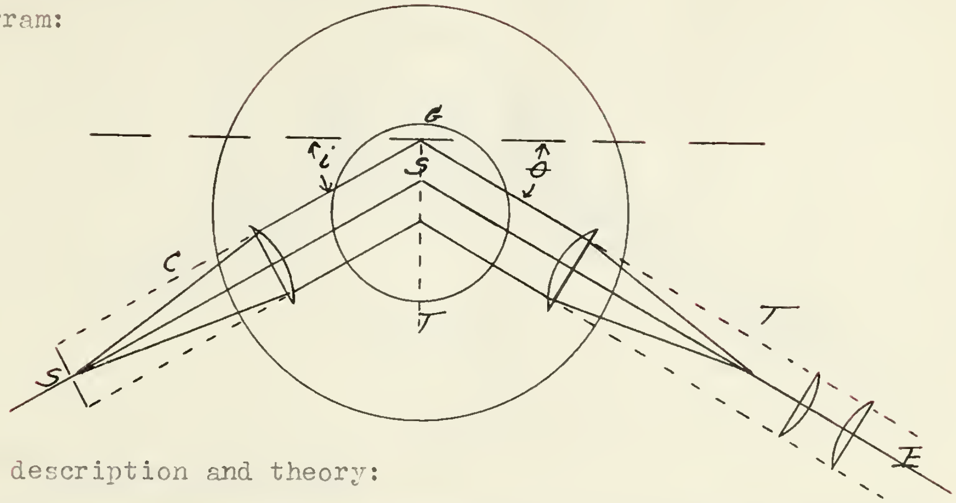
(3) The index of refraction of the glass of which the prism is made, for sodium light, may be calculated from the above data by using the following equation;

$$M = \frac{\sin 1/2\,(A+\theta)}{\sin \frac{A}{2}}$$

B. Gratings

1. Plane transmission gratings

Diagram:

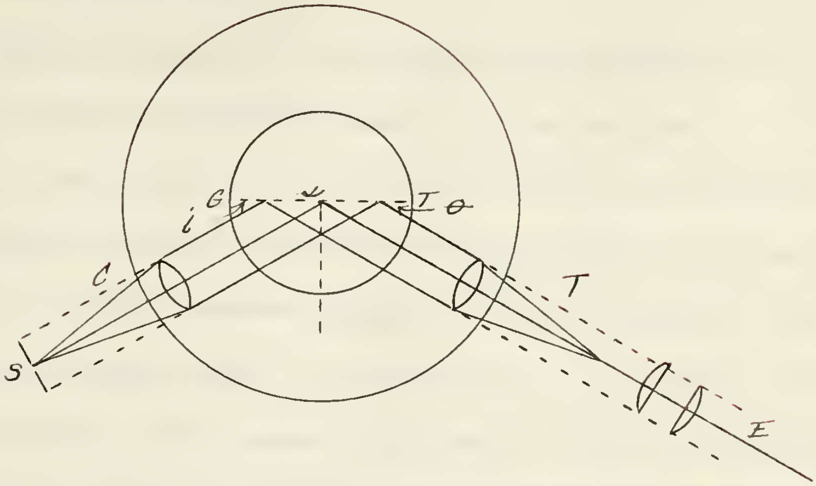


General description and theory:

A grating consists of a large number of very narrow parallel openings placed close together. These are simply obtained by ruling lines with a diamond point on a glass plate. The general theory of the order of spectrum formed is explained below: the grating receives a beam of light and each of the openings will constitute a separate source of disturbance to the medium on the opposite side of the plate. These separate sources form new wave fronts, the angle of defraction being dependent upon the distance between spaces and the wave length of light. As each wave length of light has a different amount of defraction because of differences in wave length, the grating produces dispersion. In the same way various orders of the spectrum may exist for different wave fronts dependent on differences of path. The general equation for the grating may be stated as $m\lambda = b\,(\sin i + \sin \theta)$

II. Plane reflection grating

Diagram:



General description and theory

The theory of production of the spectra follows directly from that of the plane transmission grating, the source of light acting as though it came from behind the grating. The resultant general formula for all spectra is $n\lambda = s\,(\sin i \pm \sin \theta)$, depending on whether the diffracted rays are on the same side of the normal as the incident rays or on the opposite side.

III. Concave reflection gratings

The Rowland Mount.

Diagram:

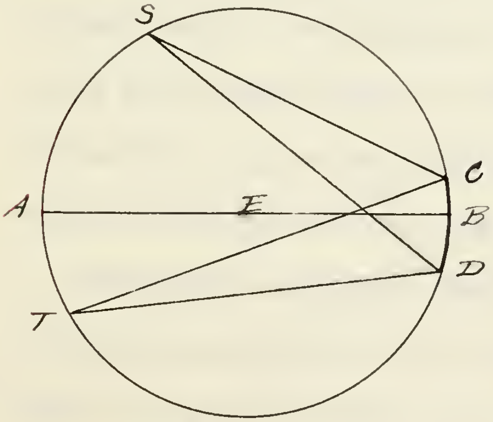


Fig 1

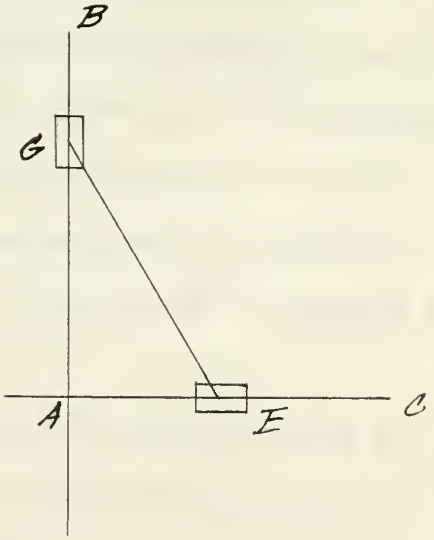


Fig 2

GENERAL DESCRIPTION AND THEORY:

In 1881 Rowland conceived the idea of ruling gratings on a spherical mirror of speculum metal. In the previous examples in dealing with the spectra produced by gratings the use of lenses has been assumed for bringing the diffracted rays to a focus, but with a concave grating these are dispensed with, because the grating itself, being ruled on a spherical mirror, focuses the rays and produces the spectra. The mathematical properties of the concave grating Rowland has completely investigated, and the instrument has proved to be one of the greatest inventions ever made in spectroscopy.

The most important property of this grating, shown by Rowland, is that if the source of light, i.e. the slit, and the grating, be placed on the circumference of the circle which has the radius of curvature of the grating as diameter, the spectra will always be brought to a focus on this circle. For example, let AB in Fig. I be the radius of curvature of the grating CD; the circle AFBG is drawn with radius $\frac{AB}{2}$, that is, with E as centre. Then if the slit be placed on the circumference of this circle, for example at F, the spectra will be formed round the circumference, so that if an eyepiece be placed at G the spectra will be seen in perfect focus, and may be examined by moving the eyepiece round the circle.

Furthermore, Rowland showed that great advantages accrue if observations are made on the spectrum normal to the grating. From the plane reflection grating above

$$\lambda = b(\sin i \pm \sin \theta),$$

GENERAL DESCRIPTION AND THEORY Cont:

and therefore when the spectrum is observed directly normal to the grating- $\lambda = b \sin i$, because $\theta = o$.

If now the eyepiece be moved a very small distance to one side of the normal, then -- $\lambda + C = b \sin i \pm b \sin \theta$, where C is the small change in wave-length observed, and θ is the angular distance through which the eyepiece has been moved. It follows that C is proportional to $b \sin \theta$, and therefore to θ itself, because b is a constant and the sines of small angles are proportional to the angles themselves. But the angle θ is proportional to the linear distance through which the eyepiece was moved, and therefore it follows that the linear distance through which the eyepiece moves is proportional to the change in wave-length observed, or, in other words, for small distances on each side of the normal the spectrum is itself normal. This fact, which is true, of course, for every grating, both flat and concave, is most important, resulting as it does in the observation of truly normal spectra, for in the mounting adopted by Rowland for his concave grating the eyepiece of photographic plate is automatically kept in a position normal to the grating.

AB and AC in Fig. 2 are two girders rigidly fastened to supports, and they carry rails which are accurately adjusted at right angles to one another. On each of these rails runs a carriage, G and E, these two carriages being joined by a beam, GE. This beam is fitted to the carriage in each case by a vertical bearing so as to allow the carriages to move along their rails. The slit is then set up

GENERAL DESCRIPTION AND THEORY Cont:

over the intersection of the rails at A, the grating at G, and the eyepiece or photographic plate at E. Under these circumstances it is evident that wherever the grating and eyepiece may be placed, the circle having GE as diameter always passes through the three points G, A, and E.

Furthermore, in order that normal spectra may be always observed, the grating is placed normal to the direction GE, and therefore, by construction, E is made to coincide with the centre of curvature of the grating. When a photographic plate is employed in place of the eyepiece at E, it is necessary that it be bent to fit the circular focal curve, when it becomes possible to photograph a considerable portion of the spectrum normal.

Rowland pointed out that a second very valuable property also results from the above mounting of the grating, and on this he based his method of relative wave-length determination. In Rowland's special case, as above, the equation of wave-length is simplified to-

$$n\lambda = b \sin i,$$

when observations are made normal to the grating.

It follows, therefore, that for one particular position of the slit--

$\lambda' = b \sin i$ in the first order,
$2\lambda'' = b \sin i$ " " second order,
$3\lambda'' = b \sin i$ " " third order, and so on,

where λ', λ'', λ''', etc., are the wave-lengths in the first, second and third, etc., orders.

GENERAL DESCRIPTION AND THEORY Cont:

We have therefore--

$$\lambda' = 2\lambda'' = 3\lambda''', \text{ etc.,}$$

and thus, at any position of the slit, the various orders of spectra are superposed, and the wave-lengths of each are inversely proportional to the number of the order. For example, on a wave-length of 9000 Angstrom units in the first order are superposed 4580 in the second order, and 3000 in the third, and 2250 in the fourth; on a wave-length of 6000 in the third order are superposed 4500 in the fourth order, 3600 in the fifth, 3000 in the sixth, and so on.

These two properties, namely, the normality of the spectrum and the relation of the superposed orders, enabled Rowland to measure the wave-lengths of all the lines in the solar spectrum with great accuracy relatively to the wave-length of one line. This, was the D line, the wave-length of which was adopted as 5896.156 as a mean of the best measurements. By measur'ng the lines in the various superposed orders in relation to the D line, Rowland first determined the wave-lengths of fourteen lines in different parts of the spectrum with the greatest possible accuracy, and in a similar way from these lines he determined the wave-lengths of the principal lines throughout the spectrum. He then photographed the whole normal spectrum from end to end, and from his knowledge of the wave-lengths of all the principal lines he was able to rule the scale of wave-lengths on each plate, and then to enlarge each photograph and its scale together. In this way a very large map of the whole spectrum

was obtained, with the scale attached from which the wave-length of any line could be read with great ease. The beauty of the method seemed to lie in the fact,that, though the wave-length of the D line is the basis of the scale, yet the relative accuracy of the scale throughout is exceedingly great, far greater than ever possible with spearate wave-length determinations.

Rowland was under the impression, owing to the accuracy of these measurements, that if at any time a more correct determination of the wave-length of the D line were made it would only be necessary to multiply his numbers and all determinations referred to them by some small factor. As a matter of fact, as will be seen from the sequel, the accuracy of Rowland's work is not nearly so great as was at first supposed. His measurements contain a periodic error which of course precludes any correction of the whole by multiplying by a constant factor.

III. Concave reflection gratings continued:

The Eagle's Mounting (Littrow type).

General description and theory:

As was briefly explained above, the coincidence method of wave-length measurements with concave gratings as used by Rowland has been found to be inaccurate, and consequently one of the great advantages of the Rowland method of mounting has disappeared, with the result that the disadvantages become more pronounced. This method has now been almost entirely superseded by the Eagle mounting, in which the greater advantages over the older method more than counterbalance the inconveniences. It is of the Littrow type, that is to say, the incident and diffracted rays make almost the same angle with the normal, and therefore the condition of minimum deviation is secured. Under these conditions the definition is the best possible, but of course the spectra obtained are not normal since the wave-length is found from the equation

$$n\lambda = 2\, b(\sin \frac{\phi}{2}).$$

The general condition laid down above is secured in this mounting, namely, that the eyepiece or photographic plate, the slit, and the grating lie on the circumference of the circle having the radius of curvature as diameter, but the focus is not automatically maintained as in Rowland's method. These are the two principal disadvantages of the Eagle mounting, they are not so great as might at first be thought.

General description and theory continued:

One of the great advantages of the Eagle mounting may be mentioned here, since this is concerned with the general theory of the concave grating. The astigmatism is very much less than with the Rowland mounting, and as the astigmatism necessarily means loss of light, the advantage gained is great compared with the Rowland mounting, where the loss of light, especially in the higher orders, is very considerable.

Dispersion:

From the general equation $n\lambda = s\,(\sin i + \sin \theta)$ by assuming the angle of incidence is constant and differentiating, the following formula for dispersion may be obtained: $m\,d\lambda = b\cos\theta\, d\theta$

and thus $$\frac{d\theta}{d\lambda} = \frac{m}{b\cos\theta}$$

where m is the order of the spectrum, b grating space and θ the angle of diffraction. This equation indicates that the dispersion reaches a minimum value when $\cos\theta$ is a maximum, that ia qhwn θ equals 0 , or $\cos\theta$ equals 1, causing the dispersion when a minimum to be equal to $\frac{m}{b}$ which may be easily calculated.

Resolving Power:

The resolving power of a grating may be expressed as $\frac{\lambda}{d\lambda} = mn = r$ where m is the order of the spectrum and n the number of apertures. This value of resolving power is derived from the general equation of the grating as follows:

Resolving Power Continued:

$$m\lambda = b (\sin i + \sin \theta)$$

multiplying by n $\quad mn\lambda = bn (\sin i + \sin \theta)$

differentiating $\quad mnd\lambda = bn \cos \theta \, d\theta$

and $\quad mn = bn \cos \theta \dfrac{d\theta}{d\lambda}$

but bn cos θ is the diameter of the diffracted ray = a

so that $\quad mn = \dfrac{ad\theta}{d\lambda}$

and from the prism derivation $\dfrac{ad\theta}{d\lambda} = \dfrac{\lambda}{d\lambda}$

Therefore $\quad mn = \dfrac{ad\theta}{d\lambda} = \dfrac{\lambda}{d\lambda} = r$

Applications:

Plane gratings find a great deal of use for ordinary work in wave length determination, particularly because of the excellent replicas of good gratings that may be obtained very cheaply. The plane transmission and reflection gratings are used in measuring the wave length of light when a large dispersion is required. The high order of these gratings may be used for hyperfine structure study.

There are five methods of using the plane grating in practice, each differing slightly from the other. In the first two methods the grating is set perpendicularly to the collimator or the telescope and thus either angle i or θ is made equal to zero causing the general equation given above to become $n\lambda = b \sin\theta$ where θ is the angle of deviation.

Applications continued:

In the third method the grating is not set perpendicularly to either the telescope or the collimator, and therefore the general equation is applicable.

The fourth method is that of minimum deviation, which in the case of the transmission grating the condition is readily obtained by so setting the grating that the angle of incidence equals the angle of diffraction. With a reflection grating, however, minimum deviation can only be obtained with the Littrow type of apparatus, in which the incident and refracted rays pass through the same telescope.

The fifth method is to fix the collimator and telescope firmly at some known angle to one another and to rotate the grating.

The serviceability of the concave grating greatly exceeds that of the plane grating for wave length determination. The fact that lenses are unnecessary with this grating and that the spectrum may be observed on the circumference of a circle where diameter is the radius of curvature, at once emphasizes the superiority of the concave over the plane grating. The concave grating is used, a great deal, in the study of the non-visible spectra.

The method of working, in general, with the concave grating may be illustrated by Rowland's method of coincidences which may be described as follows: photographs are taken of the D line in as many orders as the grating allows. Care is taken that the

Applications continued:

D line is in the center of the plate. On each side of the D line on the photograph will be found a number of lines in different orders, and if the order to which these lines belong be found by the use of absorbents, then their wave lengths may be approximately determined by measuring their distance from the D line on the plate, and by calculation from the scale of the instrument.

After this has been done these new lines are again photographed in different orders as before, in such a way that two, at least, of them are obtained, on every plate within the range of normality. The distance between them is then measured on the new plates, and from the approximate wave lengths found in the first place more accurate values are obtained of the scale of the instrument. This more accurate scale value is used for a recalculation of the wave length of the chosen lines on the first plate to a second and closer approximation. With this new value the scale of the instrument is again calculated from the second set of plates and so on until the limit of accuracy is reached.

C. I Michelson's Interferometer

Diagram:

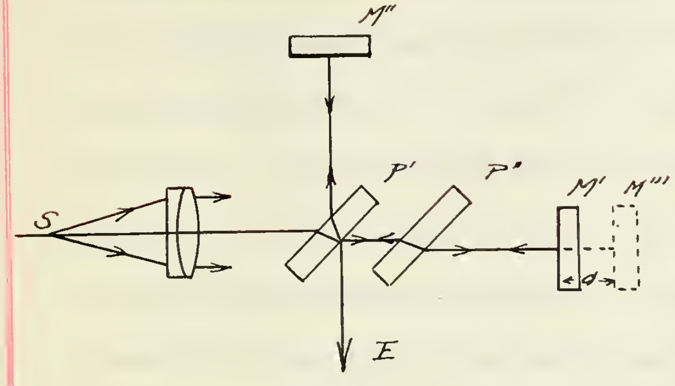


Fig 1

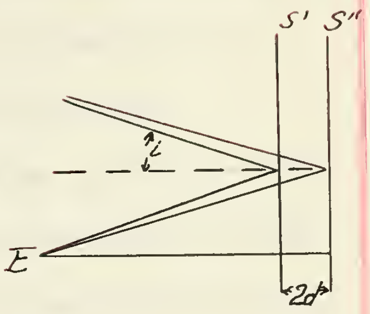


Fig 2

General description and theory:

This is a spectroscope in which a beam of light, coming from a single source is divided into two beams which traverse different paths resulting in a retardation of one beam over the other so that when they are reunited interference is produced. The theory of the instrument is as follows: the surface of the glass plate P' is half-silver, that is, the silver film is of such a thickness that about one half of the incident light is reflected. Light from the point S of an extended source falls on this surface at A and is partly transmitted to the mirror M' in part reflected to mirror M''. From these mirrors it is reflected, retraces its course and some will finally reach the eye at E. If M' and M" are at the same optical distance from S, and if they are parallel the light will appear to come from two exactly superimposed images of the source and there will be no interference. The plate P" is introduced merely to give the ray SAM' the same path in glass as ray SAM" so that the optical and geometrical paths will be the same. Now if the mirror M' be displaced to M", a distance of d the two images of the source will be displaced a distance

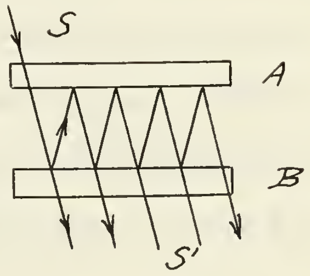
General description and theory continued:

2d but will remain parallel. The difference of path coming from two corresponding points S' and S" of these images to the eye at E is 2d cos i. Along the axis SO the difference in path is 2d and around this are circular fringes; by slowly displacing one mirror and counting the number of fringes, N, displaced across the field the distance which the mirror has moved may be determined from the relation

$$2d = n\lambda$$

C. II Fabry and Perot's Interferometer

Diagram:



General description and theory:

This interferometer consists of two half silvered parallel glass plates one movable and the other stationary. A ray of light after passing through plate A, is partially reflected and refracted at the silvered surface of B, reflected again at that of A, and so on. At each partial reflection there is also a partial transmission, and hence every ray S gives rise to a whole series S, between successive rays of which there exists a path difference equal to twice the separation of the mirrors. The fringes produced are analogous to orders in grating spectra. They are not identical, however, the differences being due to the following factors: (a) marked difference in dispersion (b) absence of single slit pattern, (c) a rapid falling of intensity of successive orders, making it possible to observe many thousands of fringes instead of a very small number. (d) all interference rings do not possess the same intensity.

Dispersion and Resolving Power:

As has already been pointed out the interferometers just described have a marked difference in dispersion and resolving power as compared with the prisms or gratings of the largest size. The resolving power of the multiple reflection interferometer is proportional to the number of components, as in the case of a grating, so that the bands are very narrow, like the spectral lines due to the latter. It is some ten times as great as with Michelson's interferometer. With these interferometers differences of path between interfering beams of several hundred thousand wave lengths have been reached. With the reference material available the derivations of the formulae of the dispersion and the resolving power of the Michelson's and the Fabry and Perot's Interferometers were unattainable for use in this paper. The formula for the resolving power of Michelson's interferometer is given by Michelson in his "Studies in Optics" as $\frac{\lambda}{d\lambda} = \frac{D}{\lambda}$ where $d = d' - D$". W. E. Williams in his "Applications of Interferometers" gives for the value of resolving power of the Fabry and Perot interferometer $\frac{\lambda}{d\lambda} = \frac{n}{\partial n}$ where $n = \frac{2\mu t \cos\theta}{\lambda}$, n being the order of the fringe, μ the refractive index and t the thickness of the plate.

Applications of the interferometers:

(a) Measurement of wave length of light.

The following description may be given of the determination of the wave lengths of the two yellow lines and the green line of mercury. The rays from both sources, mercury and cadmium, are simultaneously thrown into the apparatus, and it is absolutely indispensable that the two beams of light should be exactly superposed. Absorbents may be used to cut off the rays from either source.
It is necessary in making these measurements that the wave length of the rays to be dealt with be approximately known; these may of course, be readily found with a grating. From the approximate values of the wave lengths, one approximate value of the period of coincidence may be calculated for the two yellow rays, all the above approximations being in terms of the green ray of cadmium. Similarly, the period of the mercury green and cadmium red rays may be found in terms of the cadmium green. A calibration of the scale of the interferometer is necessary, from which at any time when a coincidence is observed the number of it can at once be obtained. After making the first series of observations, more accurate values of the period of coincidence of the rays are obtained, the wave lengths of which are to be determined, and from these the tables of coincidences may be more accurately calculated. It becomes possible then to work with thicker interference layers, when still more accurate measurements may be made and so on.

Applications of the interferometers continued:

(b) Measurement of distance in terms of wave lengths.

Michelson and Benoit applied the interferometer to the determination of the length of the standard meter in terms of the wave lengths of the cadmium lines, these rays especially being chosen on account of their width being so small; they were found to be the most homogeneous of any rays examined. A description of the method would take up too much space, and it must suffice to give the final results obtained in air at 0 and 760 mm. pressure:-

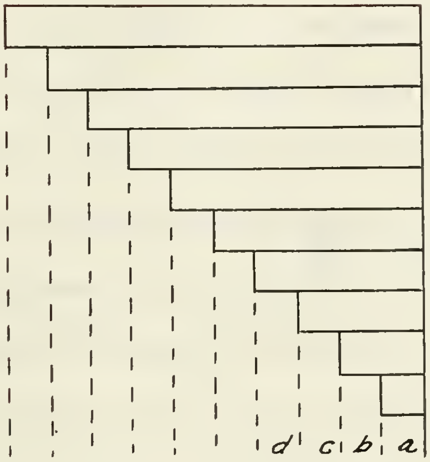
Red line	1 metre	=	1553163.5 λ,	therefore	λ = 6438.4722	tenth metres.
Green "	1 "	=	1900249.7 λ,	"	λ = 5085.8240	" "
Blue "	1 "	=	2083372.1 λ,	"	λ = 4799.9107	" "

(c) The study of fine structure in spectroscopy; visibility curves.

If in the Michelson Interferometer described above the mirror be so set that there is no difference in path between the two pencils, the illumination obtained in the telescope will of course be a maximum. If however, the mirror M" be moved away 1 mm., a difference of path will be set up of 2 mm., and a series of fringes will be seen. Michelson estimates the "visibility" of these fringes, the visibility being found from the expression $V = \frac{I' - I''}{I' + I''}$ where I' is the intensity at the center of a bright band and I" the intensity at the center of the adjoining dark band. This visibility is determined for each successive shift of the mirror M" 1 mm. outwards, and the values thus obtained are plotted on a curve. This visibility curve has various forms depending upon the distribution of light in the radiation examined, and the actual structure of the spectrum "line" can be elucidated by studying the curve obtained.

D. Michelson's Echelon Transmission Grating:

Diagram:

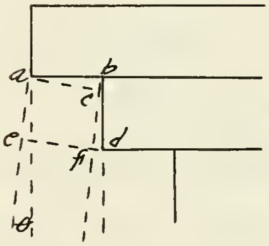


General description and theory:

This grating is made by setting together a series of perfectly parallel glass plates of equal thickness which decrease in size by an equal amount. The beam of parallel light is incident normally on the top largest plate, and the rays after passing through are brought to a focus by a lens. The pencils coming from the different plates of the grating are retarded upon one another by reason of the difference in the velocity of the light through the glass and air, and thus interference is set up at the focus of the lens and a spectrum is produced. The relative retardation between the rays from the two extreme plates is very great, because of the great difference in path through glass and air of each ray, and therefore the order of the spectrum obtained is very high. Since the direction in which the spectra are obtained is in the direction of propagation of the light the brightness of the spectra is very great.

Dispersion and Resolving Power:

The dispersion of this grating of a given glass for light of a given wave length is independent of the number of steps and varies directly as the thickness of the glass plates and inversely as the breadth of aperture of each element. The following equation gives the dispersion of an Echelon grating $\frac{d\theta}{d\lambda} = \frac{bt}{s\lambda}$ where s is the breadth of an aperture, b represents the coefficient which is entirely a function of the glass employed and may be calculated from its optical constants (it lies between 0.5 and 1.0 for most glasses), t the thickness of each plate, and the derivation of this equation is as follows: referring to the accompanying diagram



$$m\lambda = \mu bd - ae = \mu t - ae$$

$$ae = bf - bc = bd\cos\theta - ab\sin\theta$$

Therefore $m\lambda = \mu t - t\cos\theta + \sin\theta$

but θ is so small that $\cos\theta = 1$ and $\sin\theta = \theta$

so that $m\lambda = (\mu - 1)\, t + s\,\theta$ #1

differentiating with respect to λ, m being constant

$$m d\lambda = t d\mu + s d\theta$$

$$\frac{d\theta}{d\lambda} = \frac{1}{s}\left(m - t\frac{d\mu}{d\lambda}\right)$$

substituting for m its approximate value $\frac{(\mu-1)t}{\lambda}$ from #1

$$\frac{d\theta}{d\lambda} = \frac{t}{s\lambda}\left[(\mu-1) - \lambda\frac{d\mu}{d\lambda}\right]$$

$$\frac{d\theta}{d\lambda} = \frac{tb}{s\lambda} \quad \text{where } b = \left[(\mu-1) - \lambda\frac{d\mu}{d\lambda}\right]$$

Dispersion and Resolving Power Continued:

The resolving power is proportional to the total thickness of glass traversed, and for a given wave length is independent of the thickness of plates or width of steps. The following is the derivation of the value for resolving power of an Echelon grating

$$d\theta = \frac{\lambda}{a}$$

Therefore $d\theta = \frac{\lambda}{ns}$ and substituting the value of $d\theta$ in equation for the dispersion

$$\frac{\frac{\lambda}{ns}}{d\lambda} = \frac{tb}{s\lambda} \quad \text{and} \quad \frac{\lambda}{nsd\lambda} = \frac{tb}{s\lambda}$$

$$\text{or} \quad \frac{\lambda}{d\lambda} = \frac{ntb}{\lambda} = r$$

Applications:

This grating was designed by Michelson with a view of obtaining an instrument of very high resolving power, which, at the same time, would produce spectra of great brilliancy. The chief objection to this grating lies in the small value of the separation of consecutive orders of spectra, particularly so, when lines to be examined have a considerable breadth of their own.

It is necessary in making use of an echelon grating that the light be submitted to a preliminary analysis by means of a prism, before it is allowed to enter the echelon apparatus, on account of the very small angle between two successive orders of spectra. For this purpose, the most convenient form of spectroscope is the constant deviation spectrometer. With this apparatus the eyepiece is removed and the telescope object glass so adjusted as to focus the image of

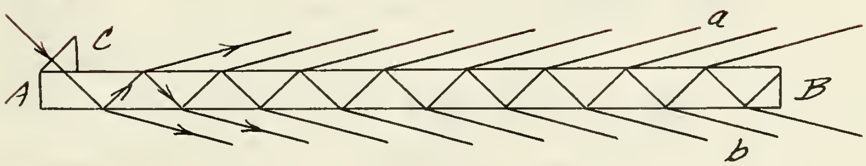
Applications continued:

the slit upon the slit of the echelon instrument; by simply turning the constant deviation prism any desired line can be brought on the echelon slit without further adjustment.

A line that is required to be examined may be brought upon the echelon slit by first passing through the constant deviation spectroscope for the preliminary analysis and different phenomena may be seen by looking through the echelon telescope. It is important to notice that on each side of the center of the field there is a dark point corresponding to the condition that $\theta = \frac{\lambda}{s}$; between these dark points is a region equivalent to twice the distance between the consecutive orders. It is this region that should be examined, and the fainter lines, lying outside the two dark points will not in any way interfere. By a slight rotation of the echelon on its vertical axis any required condition can be obtained, and a number of orders can be made to cross the field of view. It will be found during rotation that there is one position at which the direction of motion of the lines in the field is reversed; the echelon is then normal to the incident light, which is the best position for work.

E. The Lummer-Gehrcke plate:

Diagram:



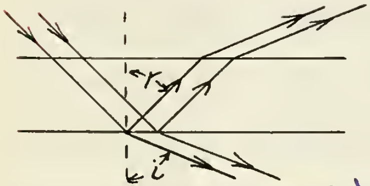
General description and theory:

This plate is especially suited to the examination of the structure of the finest spectrum lines. It consists of a glass plate AB 5.4 mm. Thick and 20 cm. long of which the upper and lower sides are made as plain as possible and parallel to one another. C is a prism cemented to the plate. A beam of light falls on C, the ray undergoes a series of reflections inside the glass plate and at each reflection some of the light is refracted into the air. Two pencils of light a and b are formed, one by rays escaping from the lower side of the plate. These two pencils enter the telescope, and the interference rings are observed at the focus of the objective.

Dispersion and Resolving Power.

The following are the derivations of the equations for the dispersion and the resolving power of the Lummer-Gehrcke plate:

The fundamental condition for reenforcement is



$2\mu t \cos r = n\lambda$ and $\sin i = \mu \sin r$

so that $n\lambda = 2t\sqrt{u^2 - \sin^2 i}$

Differentiating $n^2\lambda = 2t^2 \left(2\mu \frac{du}{d\lambda} - \sin 2i \frac{di}{d\lambda}\right)$

and Dispersion $= \frac{di}{d\lambda} = \frac{4t^2\mu\frac{d\mu}{d\lambda} - n^2\lambda}{2t^2 \sin 2i}$

By rewriting and rearranging the value for the dispersion of the Lummer-Gehrcke plate the following equation is obtained:

$$di = \frac{1}{\lambda \sin i \cos i}\left(\mu^2 - \sin^2 i - \lambda\mu\frac{d\mu}{d\lambda}\right) d\lambda$$

and making $di = \frac{\lambda}{L\cos i}$ causes corresponding $d\lambda$ to be smallest wave length change resolvable so that $\frac{\lambda}{d\lambda} = \frac{L}{\lambda \sin i}\left(u^2 - \sin^2 i - \lambda\mu\frac{d\mu}{d\lambda}\right) = r$

Applications:

With this instrument it is claimed that the bright interference bands are much stronger and hence, the weaker components are more readily seen. Consequently its chief use is in the study of fine structure where high resolving power is necessary. The following results were obtained by Lummer and Gehrcke in perhaps one of the most complete resolution of lines, in the case of the mercury vapor:

1. The less refrangivle yellow line, $\lambda = 5790$; a moderately broad principal line with five clearly separated satellites of different breadths and brightnesses. Two of the satellites appear to be double.

2. The more refrangible yellow line, $\lambda = 5730$; a moderately fine principal line very little brighter than a very closely situated satellite. Then come three satellites of weaker intensity, and then a broader and weaker satellite, which is probable double.

3. Bright green line, $\lambda = 5461$; a probable triple principal line, five bright and two weaker satellites, of which one appears to be double.

4. Dark green line, $\lambda = 4916$; a principal line with two or more closely situated satellites.

5. Blue line, $\lambda = 4350$; a great number of very fine and sharp satellites lying on each side of the principal line; Lummer and Gehrcke counted seven, but most probably there are more.

6. Less refrangible violet line, $\lambda = 4078$; one principal component with diffused edges and one narrow and one broad satellite.

7. More refrangible violet line, $\lambda = 4046$; a diffuse double principal line.

F. Spectrographs:

1. Quartz Spectographs:

The accuracy of wave length determination by visual means cannot be compared with that obtained by photographic methods. A further disadvantage of the visual method lies in the fact that the ultra violet region is not visible to the eye. The name usually given to an apparatus designed for spaectrum photography alone is a spectrograph. It is, of course possible to fit a photographic plate in place of the eyepiece of the telescope. It is far better however if the telescope be removed entirely and a wooden box set in its place. For work in the ultra violet a quartz prism and lens must be used as the light of short wave length cannot get through the glass. Many instruments have been devised to give convenient forms of work in the ultra violet. The Littrow type spectrograph is noted for its very accurate work in this field. The principle of the apparatus is similar to the Littrow prism spectroscope, except that the right angle prism and lens and the half prism used, are all made of quartz. Furthermore the second face of the half prism is coated with mercury amalgam and the rays are reflected back through the prism. They pass once again through the lens, and passing beneath the right angle prism are brought to a focus on a photographic plate.

II. The Fluorite Vacuum Spectrograph:

As previously stated the experimental work in the extreme ultra violet may be carried on either with a fluorite prism spectrograph or a concave grating instrument. The limit reached in wave length determination by a spectrograph depends on three factors, namely, the

The Fluorite Vacuum Spectrograph Cont.

absorption caused by the dispersing medium in the case of a prism spectrograph, the absorption due to air, and the decrease in sensitivity of the ordinary photographic plate.

The three absorption factors mentioned above all become effective at or about the same region in the spectrum, and consequently little use is, in general, served by elimination of any one without the other two. By using photographic plates with very little gelatine this absorption may be practically eliminated as the absorption is due to the quantity of gelatine. The use of fluorite prisms materially decreases the absorption of the dispersing medium. Finally the absorption due to air may be eliminated by the use of a vacuum spectrograph. These factors were admirably noted and developed by Schuman with his vacuum spectrograph the features of the instrument being presented below; in Schumans vacuum spectrograph the whole apparatus must be exhaused, and the following adjustments must be so arranged as to be controllable from <u>outside</u> the apparatus, without disturbing the vacuum; (1) rotation of the slit around the collimator axis; (2) the width of the slit; (3) the length of the slit; (4) the position of the effective slit aperture, this adjustment being for the purpose of taking adjacent spectra upon the plate for wave length determination; (5) focussing of both collimator and camera lenses; (6) adjustment for minimum deviation; (7) adjustment of the angle between camera and collimator so as to enable different regions of the spectrum to be photographed; (8) adjustment of the tilt of the plate in the camera; and (9) movement of the plate in a vertical plane.

III. The Grating Vacuum Spectrograph

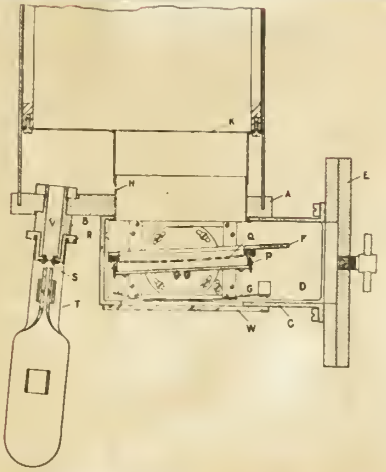


Fig. 2.

General Description and Theory:

The following description of the concave grating vacuum spectrograph, at present employed at Harvard University, was obtained from Lyman's "Spectroscopy of the Extreme Ultra-Violet."

"The end of the apparatus which containes the grating need not particularly concern us; it is closed by a vertical brass plate made airtight by a thin ring of soft wax. Most of the adjustments for focus are made by removing this plate and manipulating the grating.

The arrangement of the slit and the plate-holder end of the instrument presents some novelties of design which are illustrated in the drawing of a horizontal section (Fig.2).

The body of the spectroscope consists of a drawn brass tube of 14.9 cm. internal diameter, which carries at the grating end a flange and to which, at the other extremity, is permanently soldered the Plate A. This plate carries the slit tube B, and from it protrudes the rectangular

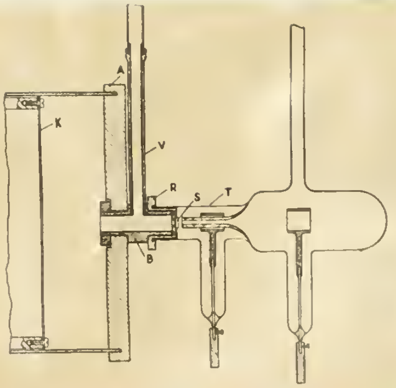


Fig. 3

brass box C, 14 cm. long, 7.8 cm. high, and 5.5 cm. wide. The brass plate D, upon which the plate-holder is mounted, slips into this box and

General description and theory continued:

rests upon its floor; a spring G, pressing on the back of the box, holds the base plate in position. The opening at the end of C is surrounded by a flange, and is closed by a plate E, the joint being made airtight by a ring of soft wax. To facilitate adjustment there is a slot in the back of C covered with a glass plate W. This window is made light-tight by a cap.

The plate-holder slides in vertical ways on the back of the vertical plate F; this plate is so mounted that it may be turned about a vertical axis, and it is also capable of motion in a direction toward the grating or away from it. These arrangements are designed to facilitate the last stages of the focusing operations.

The photographic plate is 8 cm. long and $2\frac{1}{2}$ cm. wide. A device, very similar in principle to the arrangement used in my earliest spectroscope, is operated by an electro-magnet just above C; by its means the plate-holder may be allowed to fall under gravity in its ways by successive equal steps; thus a number of exposures may be made on the same photographic plate.

The arrangements of the diaphragms in a spectroscope of this kind are of considerable importance. Light from the grating reaches the photographic plate through a slot 2 mm. wide and 7.8 cm. long in F. There is a corresponding slot in the plate A, which is furnished in turn with a narrow, rectangular box-like sleeve H, which makes connexion through a second similar sleeve with a system of diaphragms K, extending the length of the instrument (Fig. 3).

General description and theory:

The slit S has a height of 0.16 mm.; its width is usually about 0.04 mm. It can be removed from the tube B for the purpose of adjustment by melting the Khotinski cement which is employed to render the joint between B and the collar R, on which the slit is mounted, airtight. The discharge tube T is fastened to R with cement. When the spark or arc discharge is employed, a suitable container is substituted for this tube, the maximum diameter of this vessel being obviously determined by the distance between the slit and the brass box C. The arrangement of the pumping system is important; the outlet tube V is destined to remove the gas which comes through the slit from the discharge tube. The body of the spectroscope is exhausted by means of two tubes, about 1.4 cm. diameter, placed symmetrically with respect to the middle point of the instrument.

The exhaustion is effected by a mercury diffusion pump, or by the Holweck rotary pump, backed by the Trimount vacuum pump, which I have long employed; a liquid-air trap and two drying tubes are placed between the pumping system and the spectroscope.

The success of this type of instrument depends chiefly on the small dimensions of the slit and on the arrangement of the vacuum pumps whereby the body of the spectroscope is kept at a good vacuum while a pressure of several millimetres of mercury may be maintained in the vessel containing the discharge. (From Lyman's "The Spectroscopy of the Extreme Ultra-Violet.)

Summary of Best Uses of Modern Spectrometers in Wave Length Determination

1. THE PRISM

The prism spectroscope is used when only a faint source is obtainable. It is the only instrument that is practical in this case, but, the dispersion is small even with a train of prisms.

2. GRATING

(a) The plane transmission or the reflection grating must be used when a large dispersion is required.

(b) The high order of the above gratings must be used for hyperfine structure study.

3. MICHELSON'S, FABRY AND PEROT'S, THE ECHELON, AND THE LUMMER PLATE INTERFEROMETERS

Any one of these instruments may be used in the study of fine structure, where high resolving power is necessary.

4. GENERALLY FOR ALL INSTRUMENTS THE FOLLOWING RULES MAY BE OBSERVED

(a) In the study of the visible spectrum, the lenses may be of glass.

(b) In the study of the non-visible spectrum, lenses of quartz must be used.

(c) A concave grating may be substituted for #2 above.

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